1.4 Quadratic Functions – Starter Question

Show that $\dfrac{\sqrt{25k} - \sqrt{9k}}{\sqrt{k}}$ where k is an integer, is also an integer.

$$¿\frac{5\sqrt{k} - 3\sqrt{k}}{\sqrt{k}} = \frac{2\sqrt{k}}{\sqrt{k}} = 2$$

1 Express as a single power of a

$$\frac{a^2}{\sqrt{a}}$$

where $a \neq 0$

Circle your answer.

[1 mark]

a^1 $\quad$ $a^{\frac{3}{2}}$ $\quad$ $a^{\frac{5}{2}}$ $\quad$ a^4

1 Simplify $\dfrac{(a^4b)^{\frac{5}{2}}}{(a^3b^{\frac{1}{2}})^{-3}}$

Circle your answer.

[1 mark]

$a^{19}b$ $\quad$ ab^4 $\quad$ ab $\quad$ $a^{19}b^4$

1 Identify the expression below that is equivalent to $e^{\frac{-2}{5}}$

Circle your answer.

[1 mark]

$\dfrac{1}{\sqrt[5]{e^2}}$ $\qquad -\sqrt{e^5}$ $\qquad -\sqrt[5]{e^2}$ $\qquad \dfrac{1}{\sqrt{e^5}}$

2 Identify the expression below which is equivalent to $\left(\dfrac{2x}{5}\right)^{-3}$

Circle your answer.

[1 mark]

$\dfrac{8x^3}{125}$ $\qquad \dfrac{125x^3}{8}$ $\qquad \dfrac{125}{8x^3}$ $\qquad \dfrac{8}{125x^3}$

3 (a) Write down the value of p and the value of q given that:

3 (a) (i) $\sqrt{3} = 3^p$

3 (a) (ii) $\frac{1}{9} = 3^q$

3 (b) Find the value of x for which $\sqrt{3} \times 3^x = \frac{1}{9}$

3(a)(i)	States correct value of p	AO1.2	B1	$p = \frac{1}{2}$
(a)(ii)	States correct value of q	AO1.2	B1	$q = -2$
(b)	Uses valid method to find x, PI	AO1.1a	M1	$\frac{1}{2} + x = -2$
	Obtains correct x, ACF	AO1.1b	A1	$x = -2.5$
	Total		**4**	

4 Show that $\dfrac{\sqrt{6}}{\sqrt{3}-\sqrt{2}}$ can be expressed in the form $m\sqrt{n}+n\sqrt{m}$, where m and n are integers.

Fully justify your answer.

[4 marks]

Q	Marking Instructions	AO	Marks	Typical Solution
4	Multiplies by $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$	AO1.1a	M1	$\dfrac{\sqrt{6}}{\sqrt{3}-\sqrt{2}} \times \dfrac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$
	Correctly evaluates denominator to get 3 – 2 or 1	AO1.1b	A1	$= \dfrac{\sqrt{18}+\sqrt{12}}{3-2}$
	Evaluates numerator, one term correct $\sqrt{18}\ or\ \sqrt{12}$ or $3\sqrt{2}\ or\ 2\sqrt{3}$	AO1.1b	A1	$\dfrac{\sqrt{18}+\sqrt{12}}{1}$
	Completes solution CAO	AO2.1	R1	$= \sqrt{9\times 2}+\sqrt{4\times 3}$ $= 3\sqrt{2}+2\sqrt{3}$
	Total		**4**	

B Algebra and functions

B3	Work with quadratic functions and their graphs; the discriminant of a quadratic function, including the conditions for real and repeated roots; completing the square; solution of quadratic equations including solving quadratic equations in a function of the unknown.

Assessed at AS and A-level

Teaching guidance

Students should:

- be able to sketch graphs of quadratics, ie of $y = ax^2 + bx + c$
- be able to identify features of the graph such as points where the graph crosses the axes, lines of symmetry or the vertex of the graph

B Algebra and functions

- be able to complete the square and use the resulting expression to make deductions, such as the maximum/minimum value of a quadratic or the number of roots

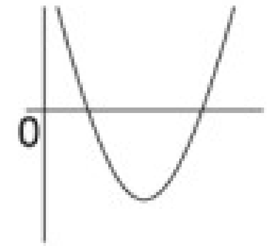
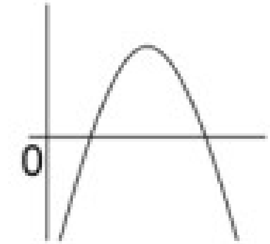
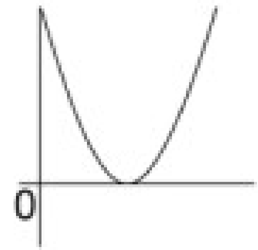
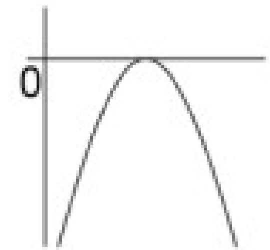
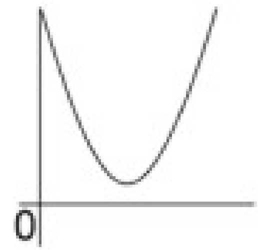
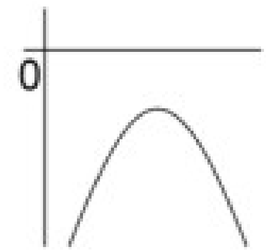
  Note: We expect students to use a calculator to solve quadratic equations and to find the coordinates of the vertex. There is no need for substitution in the quadratic formula or completing the square to justify solutions. We expect an understanding of quadratic functions, but the routine solution of equations is not in itself part of this understanding.

- be able to solve quadratic equations in a function of the unknown, where the function may be, for example, trigonometric or exponential.

Note: quadratic equations may arise from problems set in a variety of contexts taken from mechanics and statistics.

B Algebra and functions

- know and use the following:

$b^2 - 4ac > 0$	Distinct real roots		
$b^2 - 4ac = 0$	Equal roots		
$b^2 - 4ac < 0$	No real roots		

Note: a quadratic described as having real roots will be such that $b^2 - 4ac \geq 0$

1.4 Quadratic Functions

Example 1

Find the solutions of the quadratic equation $6x^2 + 17x + 7 = 0$ by factorisation.

We can use the calculator function to find the solutions!

fx-991-EX
Menu A, polynomial, degree 2

fx-991-CW
Home
Equation

1.4 Quadratic Functions

xample 2

y completing the square, find all the solutions of

$$x^2 - 14x + 33 = 0$$

1.4 Quadratic Functions

Example 3

By completing the square, find all the solutions of $4 - 3x^2 - 6x = 0$

1.4 Quadratic Functions

Example 5

Solve the equations

a $x - \dfrac{5}{x} = 4$

b $\dfrac{9}{5 - x} - 1 = 2x$

1.4 Quadratic Functions

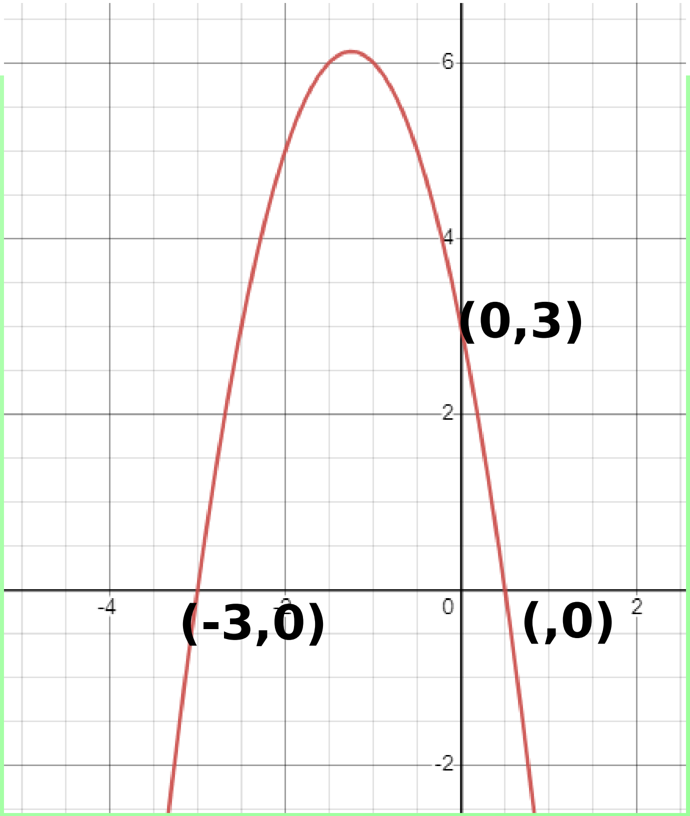
Example 6

a Find the coordinates of the turning point of the curve with equation $y = 3 - 5x - 2x^2$.

b Sketch the curve $y = 3 - 5x - 2x^2$, showing the coordinates of any points of intersection with the coordinate axes.

From calc:

Turning point:



1.4 Quadratic Functions

For each of the following quadratics, can you factorise it (if possible), complete the square, find the turning point, sketch it and give the equation of the line of symmetry:

$$x^2 - 4x + 3 \qquad x^2 - 4x + 4$$

$$x^2 - 5x + 4 \qquad x^2 - 4x + 7$$

Extension

show that the roots of the equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

1.4 Quadratic Functions

etch $f(x) = x^2 - 4x + 3$

oots of an equation are the solutions of f(x

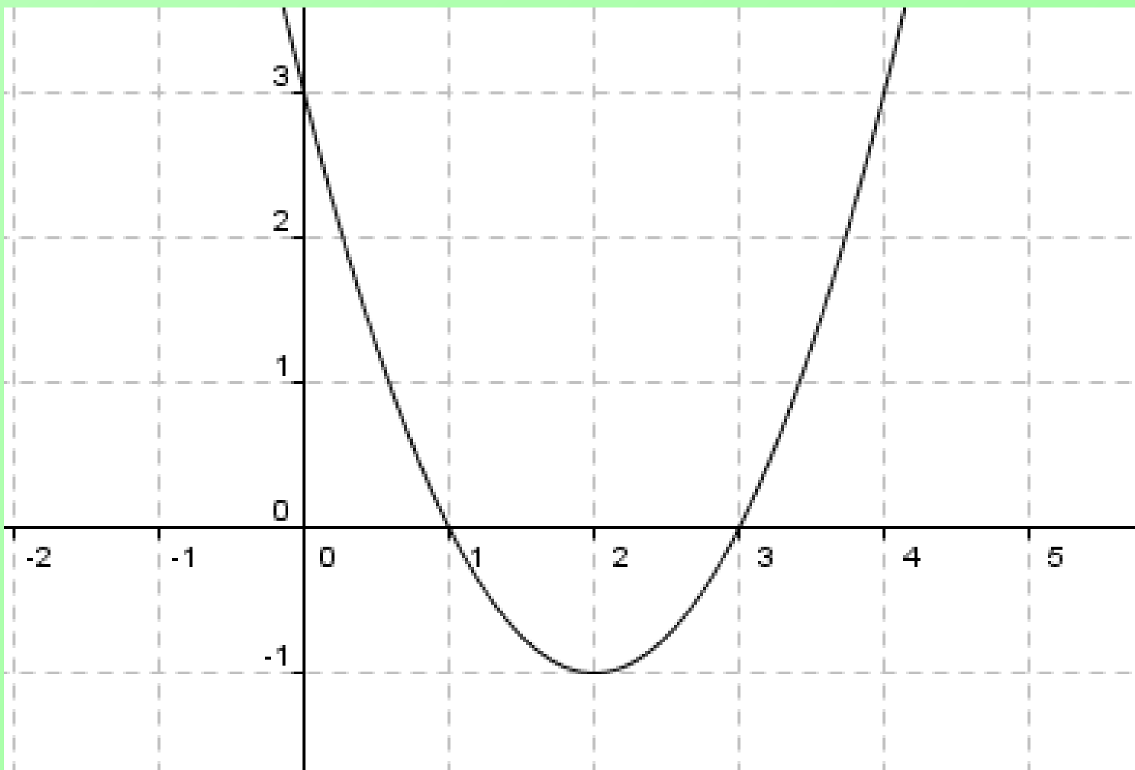
an find the roots of an equation algebraically

sing its graph. ie: Solve $x^2 - 4x + 3 = 0$

f(x) = x^2 - 4x + 3

This function has two roots.

One root is when $x = 1$ and the other root is when $x =$



1.4 Quadratic Functions

The formula for solving a quadratic equation is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

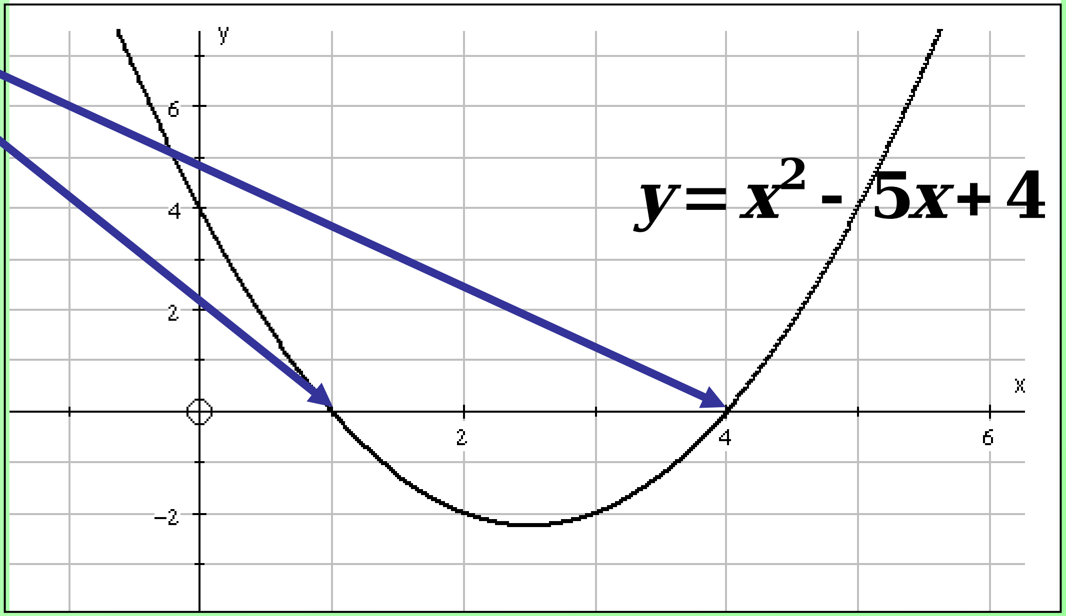
The part $b^2 - 4ac$ is called the DISCRIMINANT

We can use the value of the discriminant to indicate the type of roots we are expecting.

1.4 Quadratic Functions

Consider the graph of $y = x^2 - 5x + 4$

The roots of the equation are at the points where $y = 0$ **($x = 1$ and $x = 4$)**



The discriminant $b^2 - 4ac = 25 - 16$

$= 9 \quad > 0$

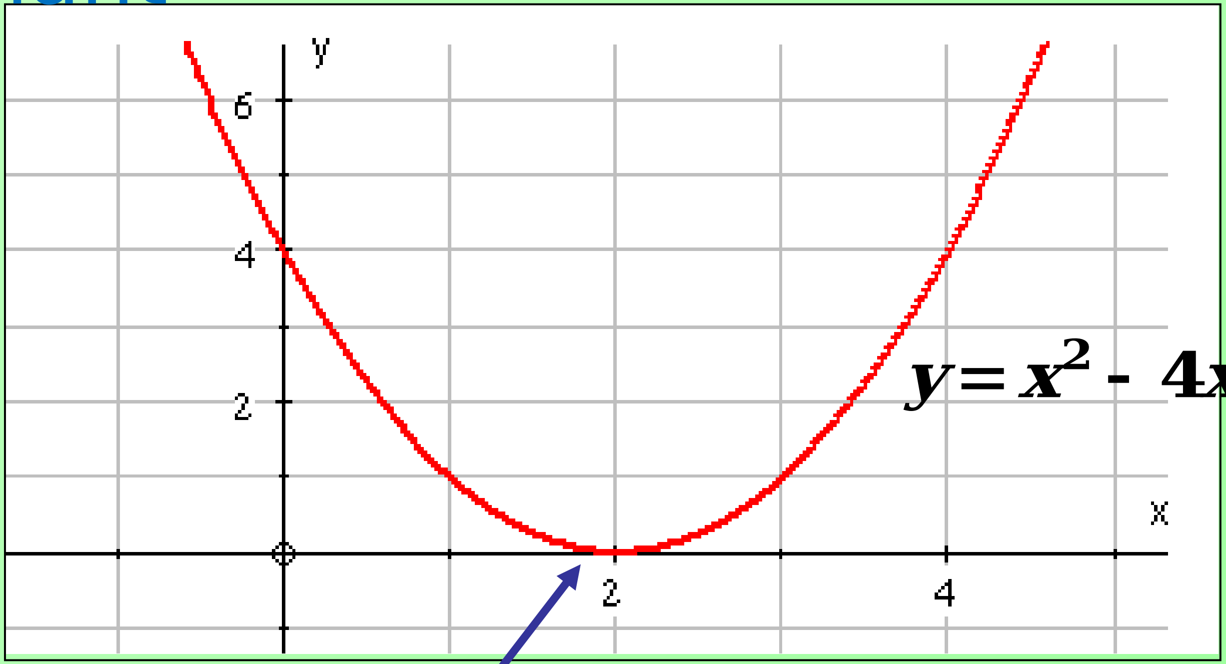
The roots are real and distinct (different)

1.4 Quadratic Functions

Consider the graph of $y = x^2 - 4x + 4$

The discriminant

$$b^2 - 4ac = 16 - 16 = 0$$



The roots are real and

(x = 2)

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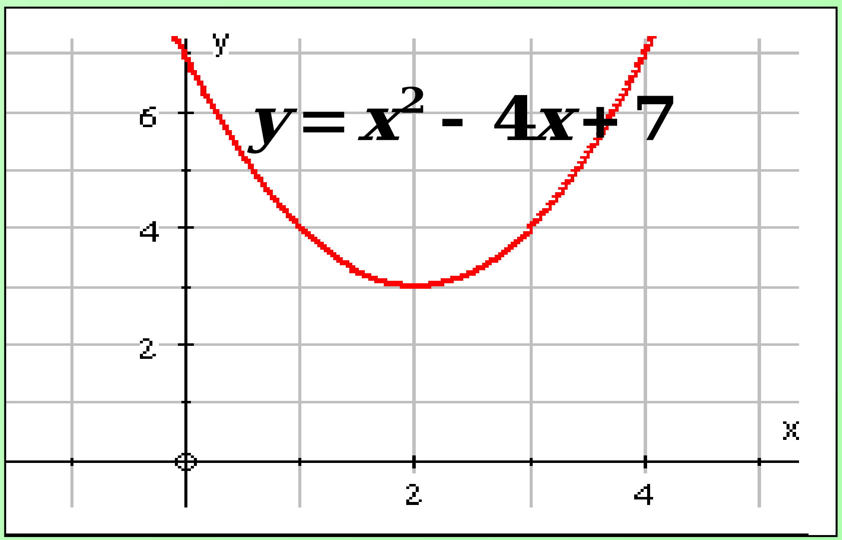
1.4 Quadratic Functions

Consider the graph of $y = x^2 – 4x + 7$

∴ the discriminant $b^2 - 4ac = 16 - 28 = -12 < 0$



There are no real roots as the function is never equal to zero

If we try to solve $x^2 - 4x + 7 = 0$ we get $x = \frac{4 \pm \sqrt{-12}}{2}$

There are no real solutions to $\sqrt{-12}$

1.4 Quadratic Functions

The formula for solving the quadratic equation $ax^2+bx+c=0$ is

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

The part b^2-4ac is called the discriminant

$b^2-4ac>0$ **The roots are real and distinct (different)**

$b^2-4ac=0$ **The roots are real and equal**

$b^2-4ac<0$ **The roots are not real**

If we try to solve an equation with no real roots, we will be faced with the square root of a

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1.4 Quadratic Functions

Examples

1. If $x^2 - 3x + a = 0$ has real distinct roots, find the possible values of a.

Find the possible values of the constant a, given tha
quation $ax^2 + (8 - a)x + 1 = 0$ has a repeated root

Show that $x^2 + 8x + 25 = 0$ has no real solutions.

Worksheet Qs